Academic mathematics is principally concerned with the exact solution of ideally posed problems: practical mathematics requires mainly the approximate solution of problems deriving from models of acknowledged imperfection. The insistence on rigor necessary for the former is more often than not a positive hindrance when approaching the latter. In this book, which is written at the high school or freshman level, an attempt is made to acquaint the mathematician who wishes to be useful, with the facts of his life, not by brutal confrontation after he has taken his degree, but by preparation for them at an earlier age.

The book deals mainly with the solution of equations. By means of simple examples (a stone falling down a well, Achilles and the tortoise), the way in which equations arise in Physics and Engineering is illustrated. Iterative methods (the method of chords, Newton's method, etc.) are then discussed; their motivation is explained with the help of numerous diagrams, and some conditions for convergence are derived.

The standard of exposition is extremely high, and the book is attractively produced.

In view of the level at which the material is presented this book is hardly of interest to the research numerical analyst, nor will it command the direct attention of those teaching numerical analysis, but for the enterprising student and the inquisitive layman it is certainly a welcome addition to the literature.

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98[X].—Calvin H. Wilcox, Editor, *Asymptotic Solutions of Differential Equations and Their Applications*, John Wiley & Sons, Inc., New York, New York, 1964, x + 249 pp., 23 cm. Price $4.95.

This book consists of the transactions of the symposium dedicated to Professor Langer and held at Madison, Wisconsin, May 4–6, 1964. Survey articles, as well as detailed presentation of recent results, are included. A careful reading of the book yields a very good idea of the methods of obtaining asymptotic solutions of differential equations, as well as their tremendous importance in the applications. Each article also contains a very good bibliography. The authors of the articles are: Clark, Erdélyi, Kazarinoff, Lewis, Lin, McKelvey, Olver, Sibuya, Turrittin, and Wasow.

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99[X, Z].—A. V. Balakrishnan & Lucien W. Neustadt, Editors, *Computing Methods in Optimization Problems*, Academic Press, New York, 1964, x + 327 pp., 24 cm. Price $7.50.

This book is the Proceedings of a conference on Computing Methods in Optimization Problems held at UCLA in January, 1964. The papers appearing in this volume will be reviewed individually, and, by necessity, these reviews must be brief.

In the first paper, entitled "Variational theory and optimal control theory," by Magnus R. Hestenes (pp. 1–22), a general problem in optimal control is formulated and is shown to be equivalent to the problem of Bolza. This paper is an outline of the approach used by Pontryagin to establish first-order conditions. The author was one of the first to see the importance of optimal control problems in variational theory and was one of the first to formulate a general control problem.

The second paper, entitled "On the computation of the optimal temperature profile in a tubular reaction," by C. Storey and H. H. Rosenbrock (pp. 25–64), treats the problem of selecting the temperature profile and the final time so as to maximize the yield of a reaction for which the kinetic equations are linear, with coefficients depending on the time and the temperature profile used. The results of various computational methods tried and reported on herein lead the authors to believe that effective methods would consist of direct hill-climbing followed by the gradient method in function space or, perhaps even better, by an analog in function space of the Newton-Raphson method. Neither dynamic programming nor Pontryagin's maximum principle were found to be as good as other methods.

The next paper, "Several trajectory optimization techniques," appears in two parts. Part I, by R. E. Kopp and R. McGill (pp. 65–89), consists of a discussion of numerical methods for optimizing trajectories, wherein computational algorithms and the advantages and disadvantages of various procedures are reviewed. Part II, by H. Moyer and G. Pinkham (pp. 91–105), is a discussion of, and a report on, the application of these methods to problems of minimum time, low thrust, and circle-to-circle transfer, with details relating to techniques and experience.

"A steepest-ascent trajectory optimization method which reduces memory requirements," by R. H. Hillsley and H. M. Robbins (pp. 107–133), presents a method resulting in reduced memory requirements which obviate the need for tape operations on an IBM 7090 and leads to the conclusion that in-flight use by a guidance computer of limited memory capacity appears feasible.

"Dynamic programming, invariant imbedding and quasilinearizations: comparisons and connections," by R. Bellman and R. Kalaba (pp. 135–145), treats a simple class of variational problems from the three points of view stated in the title and discusses the possibilities of combining these methods.

In "A comparison between some methods for computing optimum paths in the problem of Bolza," by F. D. Faulkner (pp. 147–157), the method of steepest descent of Bryson and Denham is compared with the author's direct method. A modification of the Bryson-Denham method is presented. All three methods are said to work satisfactorily, subject to minor computational difficulties relative, for example, to convergence. The relative merits of these methods are discussed.

"Minimizing functionals in Hilbert space," by A. A. Goldstein (pp. 159–165), contains a generalization of constructive techniques used by Kantorovich and Altman for the minimization of quadratic functionals. Examples from control theory and from approximation theory in L_1 are discussed.

"Computational aspects of the time-optimal control problem," by E. J. Fadden and E. G. Gilbert (pp. 167–192), examines convergence difficulties inherent in methods proposed for the solution of this problem and suggests remedies. An example, including computational results, is given.

"An on-line identification scheme for multivariable nonlinear systems," by H. C. Hsieh (pp. 193–210), discusses the identification problem and its solution for nonlinear systems represented by truncated functional power series.

"Method of convex ascent," by Hubert Halkin (pp. 211–239), discusses a computational procedure for the solution of a class of nonlinear optimal control problems. The method is based upon properties of the reachable set. An elementary description is followed by a detailed one giving theorems and refinements. An application of the method to the Goddard problem indicates its usefulness and power.

"Study of an algorithm for dynamic optimization," by R. Perret and R. Rouxel (pp. 241–259), is a pragmatic approach, as the authors state, to the organization of a particular class of dynamical systems. An experimental computer has been constructed, and the experimental results are reported elsewhere.

The last three papers are concerned with the use of hybrid analog-digital computation, in an attempt to use the best features of each type of computer. These papers are entitled, respectively: "The application of hybrid computers to the iterative solution of optimal control problems," by E. G. Gilbert (pp. 261–284); "Synthesis of optimal controllers using hybrid analog-digital computers," by B. Paiewonsky, P. Woodrow, W. Brunner, and P. Halbert (pp. 285–303); and "Gradient methods for the optimization of dynamic systems parameters by hybrid computation," by G. A. Bekey and R. B. McGhee (pp. 305–327). The first two of these papers use algorithms based on the theory of time-optimal control, particularly the computational methods proposed by Neustadt and Eaton. Included are descriptions of computer programs and reports of computer studies.

As a whole, this book makes a valuable contribution in presenting under one cover our, as yet, primitive knowledge on the subject. In the opinion of this reviewer, it is the best book of its kind to date.

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100[X, Z].—Howard S. Bowman, *A Nomogram for Computing* $(a + jb)/(c + jd)$ *and a Nomogram for Computing* $| a + jb |/| c + jd |$, National Bureau of Standards Technical Note 250, U. S. Government Printing Office, Washington, D. C., 1964, ii + 13 pp., 26 cm. Price 15 cents (paperback).

Since $(a + jb)/(c + jd) = [(a/d + b/c) + j(b/d - a/c)]/N$, where $N = c/d + d/c$, the real and imaginary terms of the right-hand member can be computed on the same nomogram composed of three logarithmic scales.

The absolute value α of the ratio is determined by the use of three logarithmic scales, of which two are

$$L(x) = \log x - 1 \quad \text{and} \quad R(x) = 1 - \log x,$$

and the relation $2m(y) = L(\sqrt{(1 - y)}) + R(\sqrt{(1 - 1/y)})$, where $\log \alpha = 4m(\alpha)$.

The reviewer recalls a much simpler method which was described by Jesse W. M. DuMond in "(A complex quantity slide rule," in the *Journal of the American Institute of Electrical Engineers*, v. 44, 1925, pp. 133-139.) This used a chart on a drafting table. This method has been mechanized recently in a commercial cylindrical slide rule.

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